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Irina V. Rasina, Oles V. Fesko, Oleg V. Usenko

Analytical design of controllers for discrete-continuous systems with linear control

Abstract. The study focuses on a certain kind of discrete-continuous systems (DCS): the linear hybrid DCS with state-dependent coefficients. The authors proposed a problem similar to the analytical design of optimal controllers (ADOC). For this study, we generalized the Krotov sufficient optimality conditions. The paper includes several examples.

Key words and phrases: discrete-continuous systems, sufficient optimality conditions, ADOCs problem.

2020 *Mathematics Subject Classification*: 49M30; 49N10

Introduction

In some of the widely used control processes [1–7], the definitions of controlled differential or discrete systems change over time. One class of such processes is discrete-continuous systems [2]. For this study, we generalized the Krotov sufficient optimality conditions [8, 9]. The simulation model of a discrete-continuous system is presented in [2, 10–12]. It is a two-level model. The upper level contains the definitions of individual stages of uniform continuous processes. The lower (discrete) level combines these definitions into a single process and controls the entire system to minimize the functional.

This study focuses on a certain kind of discrete-continuous systems (DCS) linearly dependent on the state and control variables. The coefficient matrix elements of this DCS depend on the upper and lower level state variables. Such systems are the closest to the so-called weakly nonlinear systems [15–17].

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Below we describe a model of a linear discrete-continuous system with state-dependent coefficients and formulate a problem similar to the well-known analytical design of optimal controllers (ADOC) [14]. To solve it, we generalized the Krotov sufficient optimality conditions [8,9]. The Riccati equation containing a matrix of the Krotov function second-order derivatives is conventionally used [8]. We should find the vector-matrix system solution for both first- and second-order Krotov function derivatives at both levels with similar reasoning. In such systems, the coefficient matrices for the above-specified derivatives (just like in the original problem) depend on the state variables at the same levels.

We proposed a possible approach to solving the problem. The resulting algorithm and some examples are shown below.

1. Model of linear discrete-continuous system with state-dependent coefficients

We consider a special case of discrete control system [9]:

$$\begin{gathered} x(k+1) = A(k,x(k))x(k) + B(k,x(k))u(k), \\ x^0(k+1) = x^0(k) + \frac{1}{2}\left((x)^T S(k,x)x + (u(k))^T Q(k,x)u(k)\right), \\ k \in \mathbf{K} = \{k_I, k_I+1, ..., k_F\}, \end{gathered} \tag{1}$$

where k is the step (stage) number, time is not necessarily physical, x is the state and u is the conrol; A and S are the $m(k) \times m(k)$ matrices; B is the $m(k) \times p(k)$ matrics and Q is the $p(k) \times p(k)$ matrics; k_I is the initial step and k_F is the final step;

$$x(k) \in \mathbb{R}^{m(k)}, \quad u(k) \in \mathbb{R}^{p(k)}.$$

On some subset of the set of linearity segments $\mathbf{K}' \subset \mathbf{K}$, $k_F \notin \mathbf{K}'$, there is a continuous system of the lower level

$$\dot{x}^c = \frac{dx^c}{dt} = A^c(k,t,x^c)x^c + B^c(k,t,x^c)u^c, \quad t \in \mathbf{T}(k) = [t_I(k), t_F(k)], \tag{2}$$

where A^c is a $n(k) \times n(k)$ matrix and B^c is a $n(k) \times r(k)$ matrix, $x^c(k,t) \in \mathbb{R}^{n(k)}$, $u^c(k,t) \in \mathbb{R}^{r(k)}$.

Set an intermediate goal objective for the system (3) on the interval $[t_I(k), t_F(k)]$ in the form of a functional:

$$\dot{x}^c = \frac{dx^c}{dt} = A^c(k,t,x^c)x^c + B^c(k,t,x^c)u^c, \qquad t \in \mathbf{T}\,(k) = [t_I(k), t_F(k)], \tag{3}$$

Here S^c and Λ^c are a $n(k) \times n(k)$ matrices, Q^c is a $n(k) \times n(k)$ matrix, and λ^c is an $n(k)$-dimensional vector. For each $k \in \mathbf{K}'$ the right-hand side operator (1) is given by

$$x(k+1) = \theta(k)x^c_F,$$

where $\theta(k)$ is an $m(k) \times n(k)$ matrix and $x^c(k, t_I) = \xi(k)x$. Here $\xi(k)$ is the given matrix. We suppose the matrices Q, Q^c to be negative definite for each k and each pair (k,t).

The solution of this two-level system is such the element $m = (x(k), u(k))$, that for each $k \in \mathbf{K}'$,

$$u(k) = m^c(k), \quad m^c(k) \in \mathbf{D}^c(k),$$

where $m^c(k) = (x^c(k,t), u^c(k,t))$ is a continuous on $t \in \mathbf{T}(z)$ process. Here $\mathbf{D}^c(k)$ is the set of permissible processes m^c satisfying the specified differential system (3).

Let us denote the set of elements m satisfying all the above conditions by $\mathbf{D}$ and call it a set of permissible linear discrete-continuous processes with state-dependent coefficients.

For the model (1), (3), we consider the problem of finding the minimum on $\mathbf{D}$ of the functional

$$I = F(x_F) = \big(\lambda(x_F)\big)^T x_F + \frac{1}{2}(x_F)^T \Lambda(x_F) x_F$$

under fixed $k_I = 0$, $k_F = K$, $x(k_I)$. Here $x_F = x(k_F)$; λ and Λ are $m(k) \times 1$ and $m(k) \times m(k)$ matrices, respectively.

Note that a heuristic approach represents the researcher's preferences for constructing an upper-level discrete model that unites uniform continuous systems operating over different time intervals. There can be more than one model. The researcher selects information about completing a step to send to the upper level and control values to send from the upper to the lower. There are no available sources about selecting the only upper-level model.

2. Sufficient optimality conditions

The considered optimal control problem is a modification of the classical ADOC problem [14]. It is also a special case of the optimal control problem for DCS with intermediate criteria [13]. Its peculiar feature is that the elements of the A, B, θ, Λ, A^c, B^c, Λ^c, S^c, Q^c and the λ vector depend on the upper and lower level process states, and are not constant.

Subsequently, we use a modification of the Krotov sufficient optimality conditions for the DCS with intermediate criteria. The conditions are formulated as a theorem.

Theorem 1. [13]. *Let there be a sequence of discrete-continuous processes* $\{m_s\} \subset \mathbf{D}$ *and functionals* φ, φ^c *such that*

(1) $\mu^c(k,t)$ *is piecewise continuous for each* k*;*

(2) $R(k, x_s(k), u_s(k)) \to \mu(k),\ k \in \mathbf{K}$;

(3) $\int\limits_{\mathbf{T}(k)} \left(R^c\left(k,t,x^c_s(t), u^c_s(k,t)\right) - \mu^c(k,t)\right)dt \to 0,\ k \in \mathbf{K}',\ t \in \mathbf{T}(k)$*;*

(4) $G^c\left(k, x^c_{Fs}, x^c_{Is}\right) - l^c(k) \to 0,\ k \in \mathbf{K}'$*;*

(5) $G(x_s(t_F)) \to l$.

Then the sequence $\{m_s\}$ *is a minimizing sequence for* I *on* $\mathbf{D}$.

Let us write down these conditions for the problem under consideration. Below we define the upper and lower level Krotov functions as

$$\varphi(k) = \psi^T(k)\,x + \frac{1}{2}x^T\sigma(k)\,x - x^0,$$
$$\varphi^c(z,t,x^c) = \psi^{cT}(k,t)x^c + \frac{1}{2}x^{cT}\sigma^c(k,t)\,x^c,$$

where $\psi(k)$ is a m-dimensional vector function, $\psi^c(k,t)$ is a n-dimensional vector function, $\sigma^c(k,t)$ is a $n\times n$-matrix, $\sigma(k)$ is a $m\times m$-matrix. We have

$$\begin{aligned}
R\big(k,x(k),u(k)\big) &= \psi^T(k+1)\Big(A\big(k,x(k)\big)x(k) + B\big(k,x(k)\big)u(k)\Big)\\
&+ \frac{1}{2}\Big(A\big(k,x(k)\big)x(k) + B\big(k,x(k)\big)u(k)\Big)^T\sigma(k+1)\\
&\times\Big(A\big(k,x(k)\big)x(k) + B\big(k,x(k)\big)u(k)\Big)\\
&+ \frac{1}{2}\big(x^T(k)S(k,x)x(k) + u^T(k)Q(k,x)u(k)\big)
\end{aligned}$$

$$-\psi^T(k)x - \frac{1}{2}x^T\sigma(k)x, \qquad k \in \mathbf{K},$$

$$\begin{aligned}
&R^c\big(k,t,x^c(k,t),u^c(k,t)\big)\\
&\quad = \big(\psi^c(k,t)+\sigma^c(k,t)x^c\big)^T\big(A^c(k,t,x^c)x^c + B^c(k,t,x^c)u^c\big)\\
&\qquad - \frac{1}{2}\Big((x^c)^T S^c(t,k,x^c)x^c + \big(u^c(k,t)\big)^T Q^c(k,t,x^c)u^c(k,t)\Big)\\
&\qquad + \dot{\psi}^c(k,t)x^c + \frac{1}{2}x^{cT}\dot{\sigma}^c(k,t)x^c, k \in \mathbf{K}',
\end{aligned}$$

$$\begin{aligned}
&G^c\big(k,x,x_F^c,x_I^c\big)\\
&\quad = -\psi^T(k+1)\theta(k)x_F^c + \frac{1}{2}\big(\theta(k)x_F^c\big)^T\sigma(k+1)\theta(k)x_F^c\\
&\qquad + \psi^T(k)x + \frac{1}{2}x^T\sigma(k)x + \psi^{cT}(k,t_F)x_F^c + \frac{1}{2}x_F^{cT}\sigma^c(k,t_F)x_F^c\\
&\qquad - \psi^{cT}(k,t_I)x_I^c - \frac{1}{2}x_I^{cT}\sigma^c(k,t_I)x_I^c, \quad k \in \mathbf{K}',
\end{aligned}$$

$$\begin{aligned}
G(x) &= (\lambda)^T x_F + \frac{1}{2}(x_F)^T\Lambda x_F + \psi^T(k_F)x_F + \frac{1}{2}x_F^T\sigma(k_F)x_F\\
&\quad - \psi^T(k_I)x_I - \frac{1}{2}x_I^T\sigma(k_I)x_I,
\end{aligned}$$

$$\mu^c(z,t) = \sup\ \{R^c(z,t,x^c,u^c)\colon x^c,u^c\},$$

$$l^c(z) = \inf\ \{G^c(k,x,x_F^c,x_I^c)\colon\ x_F^c,x_I^c\},$$

$$\mu(k) = \begin{cases} \sup\{R(k,x,u)\colon x,u\}, & k \in \mathbf{K}\backslash\mathbf{K}',\\ -\inf\{l^c(z)\colon x\}, & k \in \mathbf{K}',\end{cases}$$

$$l = \inf\{G(x)\colon\ x\}.$$

3. Solution algorithm

For brevity, we will suppress the arguments of functions whenever needed and the context is clear. Suppose that the elements of the matrices A, B, Q, S for all $k \in \mathbf{K}$ and A^c, B^c, Q^c, S^c, θ for all $k \in \mathbf{K}'$, $t \in [t_I(k), t_F(k)]$ are constant and $x^c(k,t_I) = \xi(k)x$ where $\xi(k)$ is a given $n \times m$ matrix. Note that the interval $[t_I(k), t_F(k)]$ can be divided into partial intervals, and for each of them the elements of the matrices are constant. In this case, the number of lower-level continuous subsystems and the number of the $\mathbf{K}$ set elements are increased.

According to the sufficient optimality conditions formulated above, we can find the control values providing a maximum of R and R^c functions. We get

$$\begin{aligned} R_u &= (x^T A\sigma + \psi^T)B + (B^T \sigma B + Q)u = 0, \\ R^c_{u^c} &= B^{c\mathrm{T}}(\psi^c + \sigma^c x^c) - Q^c u^c = 0. \end{aligned}$$

Then

$$\begin{aligned} \tilde{u} &= -(B^T \sigma B + Q)^{-1} B^T (\psi + A^T \sigma x), \\ \tilde{u}^c &= (Q^c)^{-1} B^{c\mathrm{T}} (\psi^c + \sigma^c x^c). \end{aligned}$$

Let us substitute the expressions for $\tilde{u}$ and $\tilde{u}^c$ into the expressions for R, R^c and rearrange them:

$$\begin{aligned} P(k,x) &= R\left(k, x\left(k\right), \tilde{u}\left(k\right)\right) \\ &= \Big(\psi^T(k+1)A - \psi(k) \\ &\qquad - \psi^T(k+1)B\big(B^T\sigma(k+1)B + Q\big)^{-1} B^T A^T \sigma(k+1)\Big)x \\ &\quad + \frac{1}{2}x^T\Big(A^T\sigma(k+1) + S - \sigma(k) \\ &\qquad - A^T\sigma(k+1)B\big(B^T\sigma(k+1)B + Q\big)^{-1} B^T A\sigma(k+1)\Big)x \\ &\quad - \frac{1}{2}\psi^T B(B^T\sigma(k+1)B + Q)^{-1} B^T \psi, \\ P^c(k,t,x^c) &= R^c\left(k,t,x^c \tilde{u}^c(k,t)\right) \\ &= (\psi^{c\mathrm{T}})A^c + \sigma^c B^c (Q^c)^{-1} B^{c\mathrm{T}} \sigma^c + \dot{\psi}^c)x^c \\ &\quad + \frac{1}{2}x^{c\mathrm{T}}(\sigma^c A^c + A^{c\mathrm{T}}\sigma^c + \sigma^c B^c (Q^c)^{-1} B^{c\mathrm{T}} \sigma^c - S + \dot{\sigma}^c)x^c \\ &\quad + \psi^{c\mathrm{T}} B^c (Q^c)^{-1} B^{c\mathrm{T}} \psi^c. \end{aligned}$$

We apply the Bellman sufficient optimality conditions [**13**] and specify the functions φ and φ^c in such a way that the constructions $P(k,x)$ and $P^c(k,t,x^c)$ do not depend on the x and x^c, respectively. Then we obtain

$$\begin{aligned} \psi(k) &= \Big(A^T - \sigma(k+1)AB\big(B^T\sigma(k+1)B + Q\big)^{-1} B^T\Big)\psi(k+1), \\ \sigma(k) &= A^T\sigma(k+1)A + S - A^T\sigma(k+1)B\big(B^T\sigma(k+1)B + Q\big)^{-1} B^T A \end{aligned}$$

for $k \in \mathbf{K}\backslash\mathbf{K}'$, and for $k \in \mathbf{K}'$

$$\dot{\psi}^c = -\big(A^{c\mathrm{T}} + \sigma^c B^c (Q^c)^{-1} B^{c\mathrm{T}}\big)\psi^c,$$
$$\dot{\sigma}^c = S - \sigma^c A^c - A^{c\mathrm{T}}\sigma^c - \sigma^c B^c (Q^c)^{-1} B^{c\mathrm{T}}\sigma^c.$$

Let us now require that the functions G, G^c are independent of the upper and lower level process states taking into account the expression for x_I^c. These requirements define the initial conditions for the system of vector-matrix equations with respect to ψ, ψ^c, σ, σ^c and the equations for $\psi(k)$, $\sigma(k)$ on the set $\mathbf{K}'$. We get

$$\psi(k) = \Big(A^T - \sigma(k+1)AB\big(B^T\sigma(k+1)B + Q\big)^{-1}B^T\Big)\psi(k+1), \qquad k \in \mathbf{K}\backslash\mathbf{K}',$$
$$\sigma(k) = A^T\sigma(k+1)A + S - A^T\sigma(k+1)B\big(B^T\sigma(k+1)B + Q\big)^{-1}B^T A, \qquad k \in \mathbf{K}\backslash\mathbf{K}',$$
$$\dot{\psi}^c = -\big(A^{c\mathrm{T}} + \sigma^c B^c (Q^c)^{-1} B^{c\mathrm{T}}\big)\psi^c, \qquad k \in \mathbf{K}',$$
$$\dot{\sigma}^c = S - \sigma^c A^c - A^{c\mathrm{T}}\sigma^c - \sigma^c B^c (Q^c)^{-1} B^{c\mathrm{T}}\sigma^c, \qquad k \in \mathbf{K}'.$$

Finally, we obtain a DCS concerning ψ, ψ^c, σ, σ^c with its initial conditions at the right endpoints of the discrete and continuous arguments. The matrices of this DCS also depend on the upper and lower level state variables. With the relations we obtained, it is possible to build a solution algorithm as follows

1. We solve the initial DCS system “from left to right" for the given $u_s(k)$, $u_s^c(k,t)$ and constant matrices A, B, A^c, B^c. In this way, we obtain the corresponding states $x_s(k)$, $x_s^c(k,t)$.
2. We solve the DCS “from right to left” with respect to ψ, ψ^c, σ, σ^c with constant matrices A, B, A^c, B^c, S, S^c, Q, Q^c.
3. We find new controls $\tilde{u}_s$, $\tilde{u}_s^c$, go to step 1.
4. The iteration process ends when $|I_{s+1} - I_s| < \epsilon$, where ϵ is the required solution accuracy.

NOTE 1. The elements of the A^c, B^c matrices can be found for $x^c = x^c(k, t_I)$ while the elements of the S^c, Q^c matrices can be found for $x^c = x^c(k, t_F)$ and do not change within the interval $[t_I(k), t_F(k)]$.

NOTE 2. For better accuracy, the intervals $[t_I(k), t_F(k)]$ can be divided into subintervals resulting in more DCS stages.

Note 3. The resulting DCS for the ψ, ψ^c vectors and the σ, σ^c matrices contains matrix Riccati equations that may have no solutions. In this case, the proposed algorithm should be further developed.

Note 4. The problem of algorithm convergence is out of the scope of this paper but the examples given below prove its validity.

Let us show the algorithm performance with some examples.

4. Computational experiments

Example 1. Let us consider an optimal control problem for a nonlinear controlled system

$$\dot{x}_1 = \frac{1}{x_2}x_1 + x_1x_2u, \qquad x_1(0) = 2,$$
$$\dot{x}_2 = x_1^3x_2 - \sqrt{x_1}u, \qquad x_2(0) = 1,$$
$$I = \int_0^2 (x_1^2x_2) + \exp(-x_1)u^2)dt + x_2^2(2).$$

By dividing the interval where the system is defined (the number of subdivisions K is variable) we can transform it into a linear DCS with state-dependent coefficients. We get where $n = 2$, $r = 1$ and the interval to be divided is $[0, 2]$. Here the matrices used in the problem statement are as follows:

$$A^c = \begin{pmatrix} \frac{1}{x_2} & 0 \\ 0 & x_1^3 \end{pmatrix}, \quad B^c = \begin{pmatrix} x_1x_2 & -\sqrt{x_1} \end{pmatrix},$$

$$S^c = \begin{pmatrix} x_2 & 0 \\ 0 & 0 \end{pmatrix}, \quad Q^c(x_1, x_2) = \exp(-x_1).$$

Since only the coefficients of these matrices are modified at each partial interval, then $\mathbf{K} = \{0, 1, 2, \ldots, k_F\}$, $\mathbf{K} = \mathbf{K}'$, and the upper-level process becomes uncontrollable. After each step, the information about the lower-level system state is sent to the upper level, and the initial conditions for the next step are defined. Thus $x = (x_1, x_2)$, $x(k+1) = x^c(k, t_F(k))$, $x_I^c(k+1, t_I(k+1)) = x(k+1)$, $\lambda = 0$, $\theta = E$, $\xi = 1$, $A = E$, $B = 0$, $S = 0$, $Q = 0$, $\psi^c = (\psi_1^c, \psi_2^c)$ and

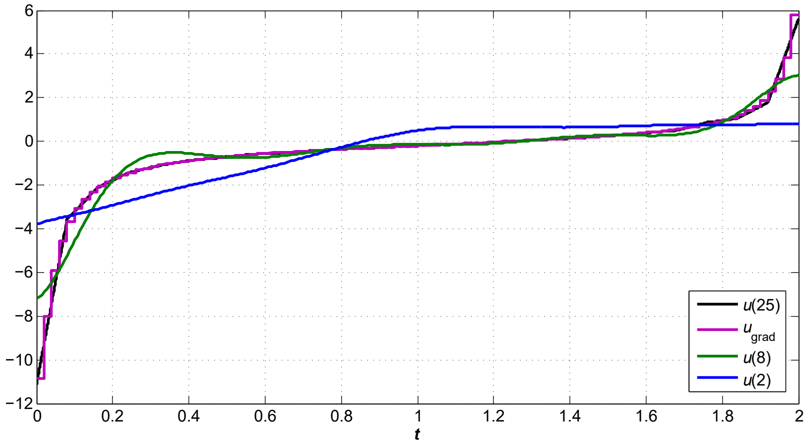

FIGURE 1. Control

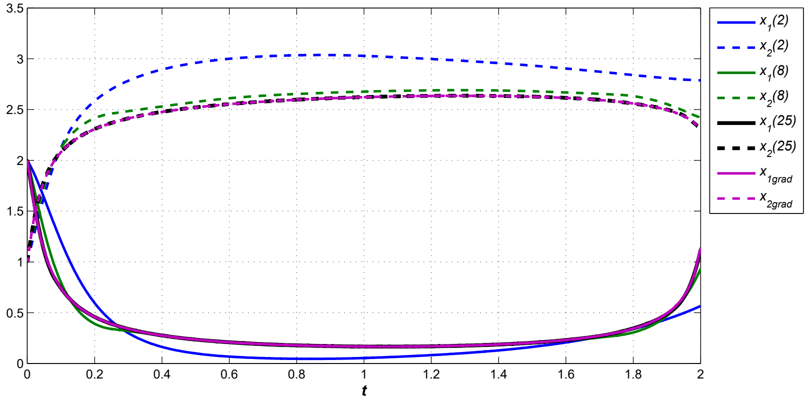

FIGURE 2. States

$$\Lambda = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^c = \begin{pmatrix} \sigma_{11}^c & \sigma_{12}^c \\ \sigma_{21}^c & \sigma_{22}^c \end{pmatrix}.$$

We analyzed cases for $k_F = 2, 4, 8, 10, 15, 25$. The results are given by Figures 1, 2. Table 1 shows the functional value vs. the number of the interval subdivisions. In each case, the problem was solved after approximately 3 iterations. The optimal control problem for a conventional

TABLE 1. Functional value vs. number of the interval subdivisions

Subdivision	Functional
2	11.14453381
4	9.62277632
8	8.95104176
10	8.84198513
15	8.72682692
25	8.67417221

controlled system was first solved by the gradient method after 5 iterations. For comparison, the figures also show the found states and the control variables. The smallest value of the functional obtained by the gradient method is $I = 8.67458$, and with the DCS algorithm it is $I = 8.67417$.

EXAMPLE 2. Consider the following linear two-stage DCS with state-dependent coefficients

First stage

$$\dot{x}_1^c = (x_1^c)^2(x_2^c - u_1^c), \quad \dot{x}_2^c = \sqrt{x_1^c}x_2^c + x_1^c u_2^c,$$

$$x_1^c(0) = 1, \quad x_2^c(0) = -1,$$

$$I^0 = \int_0^2 ((x_1^c u_2^c)^2 - x_2^c(u_1^c)^2)dt.$$

Second stage

$$\dot{x}_1^c = x_1^c \exp(-x_1^c) - 2x_1^c u_1^c, \quad I^1 = \int_2^3 \frac{(u_1^c)^2}{x_1^c}dt,$$

$$I = (x_1^c(3))^3 - 3x_1^c(3).$$

Let us transform it into the standard form. Obviously, $\mathbf{K} = \{0, 1, 2\}$. For $k = 0$ $n = r = 2$, and the lower level matrices are:

$$A^c(0) = \begin{pmatrix} 0 & (x_1^c)^2 \\ 0 & \sqrt{x_1^c} \end{pmatrix}, \quad B^c(0) = \begin{pmatrix} -(x_1^c)^2 & 0 \\ 0 & x_1^c \end{pmatrix},$$

$$Q^c(0) = \begin{pmatrix} -x_2^c & 0 \\ 0 & (x_1^c)^2 \end{pmatrix}, \quad S^c(0) = 0.$$

Table 2. Functional value vs. number of the interval subdivisions

Subdivision	Functional
2	-1.0673
4	-1.0825
10	-1.1068

At the next stage for $k = 1$ $n = r = 1$, while the lower level matrices are:

$$A^c(1) = \exp(-x_1^c), \quad B^c(1) = -2x_1^c, \quad Q^c(1) = \frac{1}{x_1^c}, \quad S^c(1) = 0.$$

It is obvious that at the top level it would be feasible to assume x as x_1^c, since the variable passes through both stages and also defined the common functional. We have $I = (x(2))^3 - 3x(2)$, $\lambda = -3$, $\Lambda = (x(2))^2$, $x(1) = x_1^c(0, 2)$, $x_1^c(1, 2) = x(1)$, $\theta = 1$, $\theta^0 = 0$, $\xi = 1$.

We analyzed the problem for $k_F = 2, 4, 10$. The results are displayed in Figures 3, 4 and Table 2. Just like in Example 1, we first found the solution with the gradient method. It took 5 iterations. With the proposed algorithm, the functional value is $I = -1.1068$. In each case, it took around 2 iterations to solve the problem. The achieved value is identical to the smallest functional value obtained with the gradient method.

The results indicate the validity of the proposed algorithm.

Conclusion

This paper presents a model of a linear discrete-continuous system with state-dependent coefficients and defines a problem similar to the well-known analytical design of optimal controllers (ADOC). By generalizing the Krotov sufficient optimality conditions, we proposed a solution algorithm that finds the first- and second-order Krotov function derivatives at both upper and lower levels. To determine the derivatives, we obtained a DCS containing the Riccati equations at both levels with respect to the second-order derivatives with their initial conditions at the right endpoints of the discrete and continuous arguments. The matrices of this DCS also depend on the upper and lower level state variables.

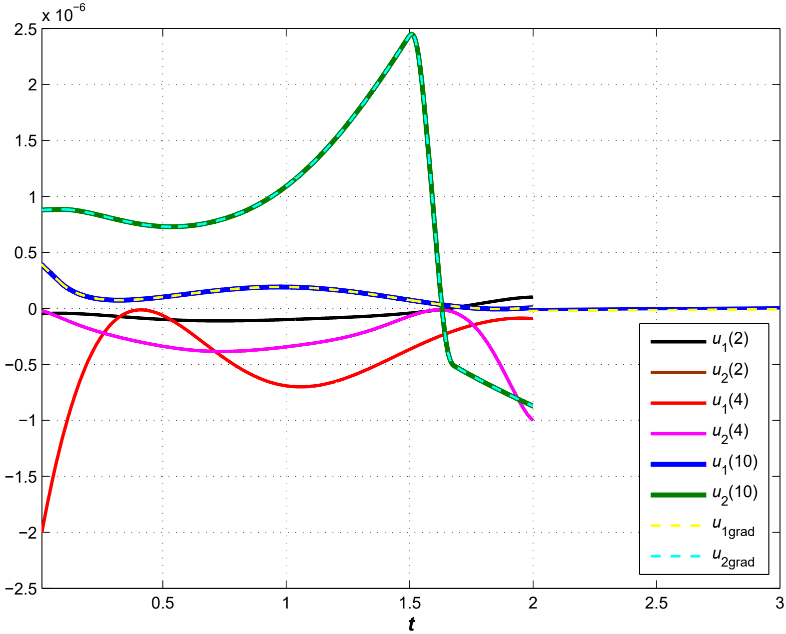

FIGURE 3. Control

The paper also includes the estimations performed with the proposed algorithm. They prove its validity. Previously, each example for comparative analysis was solved by the gradient method, while the gradient method required a larger number of iterations (in the Example 2, by one). Therefore, the complexity of the proposed algorithm does not exceed the gradient method, but it can provide a smaller value of the functional (Example 1). The explanation is the presence in conjugate systems of the upper and lower levels of equations for both the first and second derivatives of the Krotov functions. At the same time, the gradient method uses only the first derivatives of these functions.

References

[1] S. V. Emel'yanov (ed.). *Theory of Systems with Variable Structures*, Nauka, M., 1970 (in Russian), 592 pp. ↑121

[2] V.I. Gurman. "Theory of optimum discrete processes", *Autom. Remote Control*, **34**:7 (1973), pp. 1082–1087. ↑121

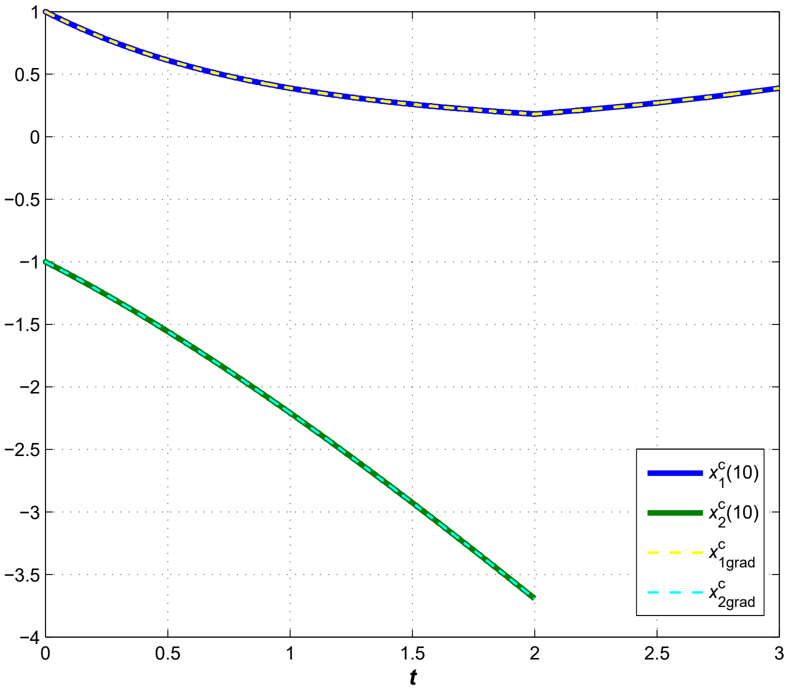

FIGURE 4. States

[3] S.N. Vassilyev. "Theory and application of logic-based controlled systems", *Proceedings of the International Conference Identification and Control Problems*, Institute of control sciences, M., 2003, pp. 23–52 (in Russian). ↑121

[4] A.S. Bortakovskii. "Sufficient optimality conditions for control of deterministic logical-dynamic systems", Informatika, Ser. Computer Aided Design, vol. **2–3**, VNIIMI, M., 1992, pp. 72–79 (in Russian). ↑121

[5] B.M. Miller, E.Ya. Rubinovich. *Optimization of the Dynamic Systems with Pulse Controls*, Nauka, M., 2005, ISBN 978-5-9710-5725-3 (in Russian), 429 pp. ↑121

[6] J. Lygeros. *Lecture Notes on Hybrid Systems*, ENSIETA 2-6/2/2004, 2004, 82 pp. URL ↑121

[7] A.J. Van der Shaft, H. Schumacher, *An Introduction to Hybrid Dynamical Systems*, Lecture Notes in Control and Information Sciences, vol. **251**, Springer-Verlag, London, 2000, ISBN 978-1-84628-542-4, xiv+174 pp. doi ↑121

[8] V.F. Krotov, V.I. Gurman. *Methods and Problems of Optimal Control*, Nauka, M., 1973 (in Russian), 448 pp. ↑121,122

[9] V.F. Krotov. “Sufficient conditions for the optimality of discrete control systems”, *Dokl. Akad. Nauk SSSR*, **172**:1 (1967), pp. 18–21 (in Russian). ↑121,122

[10] V.I. Gurman, I.V. Rasina. “Discrete-continuous representations of impulsive processes in the controllable systems”, *Automation and Remote Control*, **73**:8 (2012), pp. 1290–1300. doi ↑121

[11] I. V. Rasina. “Discrete-continuous models and optimization of control processes”, *Program Systems: Theory and Applications*, **5**:9 (2011), pp. 49–72 (in Russian). URL ↑121

[12] I.V. Rasina. “Iterative optimization algorithms for discrete-continuous processes”, *Automation and Remote Control*, **73**:10 (2012), pp. 1591–1603. doi ↑121

[13] I.V. Rasina. “Discrete-continuous systems with intermediate criteria”, *International Conference on Computational Mechanics and Modern Applied Software Systems*, CMMASS’2017, Moscow Aviation Institute, M., 2017, pp. 699–701 (in Russian). ↑124,126

[14] A. M. Letov. “Analytical design of controllers, II”, *Avtomat. i Telemekh.*, **21**:5 (1960), pp. 436–441 (in Russian). ↑122,124

[15] R. Gabasov, A. I. Kalinin, F. M. Kirillova, L. I. Lavrinovich. “On asymptotic optimization methods for quasilinear control systems”, *Trudy Inst. Mat. i Mekh. UrO RAN*, **25**:3 (2019), pp. 62–72 (in Russian). doi ↑121

[16] Y. E. Danik, M. G. Dmitriev, D. A. Makarov. “An algorithm for constructing regulators for nonlinear systems with the formal small parameter”, *Journal of Information Technologies and Computing Systems*, 2015, no. 10, pp. 35–44 (in Russian). ↑121

[17] Yu.E. Danik, M.G. Dmitriev. “A comparison of numerical algorithms for discrete-time state dependent coefficients control systems”, 21st International Conference on System Theory, Control and Computing (ICSTCC) (19–21 Oct. 2017, Sinaia, Romania), pp. 401–406. doi ↑121

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About the authors:

Irina Viktorovna Rasina

Doctor of Physical and Mathematical Sciences, Chief Researcher Research Center for Systems Analysis of the Institute of Software Systems named after A.K. Ailamazyan RAS, specialist in the field of modeling and control of hybrid systems, author and co-author of more than 100 articles and 5 monographs

0000-0001-8939-2968
e-mail: irinarasina@gmail.com

Oles Vladimirovich Fesko

Researcher, Research Center for System Analysis of A. K. Ailamazyan Institute of Program Systems of RAS, Ph.D. Area of interest: programming, numerical methods

0000-0002-9329-5754
e-mail: oles.fesko@hotmail.com

Oleg Valerievich Usenko

Lecturer at the Physics Department of Irkutsk State University and the Regional State Budgetary Professional Educational Institution of Social Services Irkutsk Rehabilitation College, author and co-author of 21 publications, area of interest — numerical optimization methods, programming

0000-0001-6291-8165
e-mail: o.v.usenko@gmail.com

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