ISSN 2079-3316 PROGRAM SYSTEMS: THEORY AND APPLICATIONS vol. 13, No 3(54), pp. 17–27
Research Article ARTIFICIAL INTELLIGENCE, INTELLIGENT SYSTEMS, NEURAL NETWORKS

UDC 531.383:519.876.5
10.25209/2079-3316-2022-13-3-17-27

Simulation of a multifunctional micromechanical gyroscope

Lyalya Uzbekovna **Bakhtieva**[1], Vladimir Mikhailovich **Bogolyubov**[2],
Maxim Dmitrievich **Tumakov**[3]

[1,3] Kazan Federal University, Kazan, Russia
[2] Kazan National Research Technical University named after A. N. Tupolev (KAI), Kazan, Russia
[1✉] *lbakhtie@yandex.ru* *(learn more about the authors on p. 26)*

Abstract. The possibility of constructing a multifunctional inertial navigation device based on a hybrid-type modulation micromechanical gyroscope is considered. A mathematical model of the device ("heavy" gyroscope) as a high-quality three-dimensional oscillatory system is constructed. It is numerically shown that, under certain conditions, the reaction of the system to the motion of an object has, along with precession, the observed nutation, which carries information about the linear motion of the gyroscope base. It is noted that the possibility of measuring linear accelerations is ensured by the presence of a small symmetrical distance between the axes of the elastic suspension relative to the center of mass of the sensing element.

The results obtained make it possible to implement a two-component angular velocity meter and a two-component linear acceleration meter in one device.

Key words and phrases: mathematical model, vibration, micromechanical system

2020 *Mathematics Subject Classification:* 70-10; 70B15

For citation: Lyalya U. Bakhtieva, Vladimir M. Bogolyubov, Maxim D. Tumakov. *Simulation of a multifunctional micromechanical gyroscope* // Program Systems: Theory and Applications, 2022, **13**:3(54), pp. 17–27. http://psta.psiras.ru/read/psta2022_3_17-27.pdf

Эта статья по-русски: *http://psta.psiras.ru/read/psta2022_3_5-15.pdf*

Introduction

The article presented by the authors is a continuation of a series of works devoted to the improvement of gyroscopic devices [1–4].

Despite the significant progress [5–11] in the development of micromechanical gyroscopes (MMG), problems remain that limit the accuracy of their measurements. The most important of them is the presence of a small mass of the sensitive element, which is unable to create the required value of the gyroscopic moment when measuring small values of the portable angular velocity, which imposes a requirement for an ultra-high sensitivity of the instrument's information retrieval system. The solution to this problem is associated with the trend of transition to hybrid MMGs that has emerged in recent years, combining the methods of planar technology for manufacturing an inertial mass using an electromechanical drive. Based on the hybrid MMG, it was possible to create a biaxial angular velocity sensor with slightly increased dimensions, but with a significantly increased accuracy [5–7].

Another problematic factor that worsens the metrological properties of the device is a significant level of "zero bias" associated with its manufacturing technology. In contrast to the method described in [7], the authors of this article propose to use the modulation principle for collecting and processing primary information in a mechanical circuit to solve this problem, which has proven itself well in vibrating gyroscopes [1]. A distinctive feature of the modulation MMG is that the registration of the angular rotation of the base of the device is carried out not by measuring this rotation relative to the main axis of the gyroscope, which is stationary in inertial space, but by measuring the amplitude and phase of oscillations of its rotor in a rotating coordinate system. In this case, information about the angular movement of the base is contained in the indicated parameters of the AC signal, and the presence of a constant component caused by the so-called “zero shift” does not affect the accuracy of taking MMG readings.

The mathematical model of the movement of the hybrid type modulation MMG proposed below and the analysis of the numerical results allow concluding that it is possible to significantly expand the measuring capacity of such devices.

1. Problem Statement

A two-coordinate hybrid MMG is capable of measuring the value of the base angular velocity vector lying in the plane of its sensitivity. When the device is tuned to resonance with respect to the measured angular

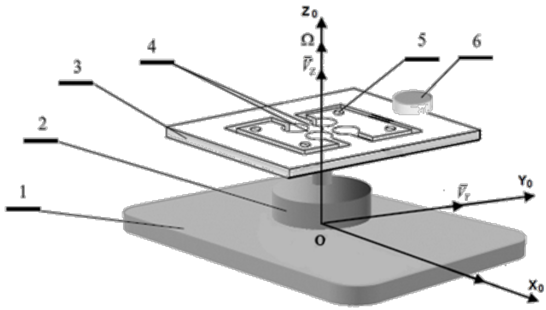


FIGURE 1. Kinematic scheme of modulating hybrid MMG-N

velocity, it works like an integrating gyroscope, since the deflection of its rotor (under conditions of low attenuation) is proportional to the angle of rotation of the base.

The purpose of this study is to develop theoretical foundations for creating a multifunctional device, which, along with the ability to measure angular deviations in two orthogonal directions coinciding with its sensitivity axes, makes it possible to measure the magnitude of the inertia force vector caused by the accelerated linear movement of the base. That is, it is required to investigate the possibility of developing an angular velocity sensor that simultaneously performs the function of an accelerometer.

The figure 1 shows a kinematic scheme of the proposed modulating micromechanical gyroscope — a hybrid type forcemeter (MMG-N), including a movable base 1, a valve drive 2, a rotor 3, an elastic suspension 4, rotor fastening elements 5, an angle transducer 6 connected to the drive shaft.

Let us show that linear acceleration can be measured due to a small symmetrical spacing of the elastic suspension axes relative to the center of mass of the sensing element.

2. Mathematical model

Figure 2 shows a general view drawing of the MMG-N rotor, the distinguishing feature of which is non-crossing of elastic supports 1, 2, creating "pendulosity" of rotor 3 due to the displacement of its center relative to the axes.

To describe the motion of a modulating gyroscope-forcemeter (MMG-N) mounted on a movable base, we introduce a coordinate system with the

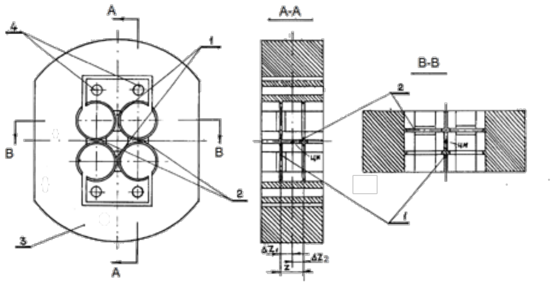


FIGURE 2. General view drawing of the rotor of the modulating hybrid MMG

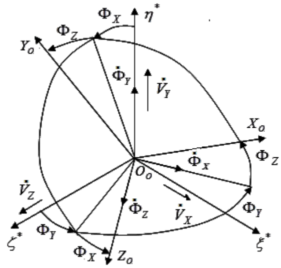


FIGURE 3. The position of the object in the base coordinate system

origin at the center of mass of the gyroscope rotor and the axes directed to fixed stars, i.e. moving translationally in inertial space (Figure refbakfig1). We associate with the moving base the coordinate system $X_0Y_0Z_0$, the axis OZ_0 of which coincides with the axis of rotation of the MMG-N rotor (figure 3).

The motion of the base of the device will be considered known, i.e. at each instant of time, the orientation of the coordinate system $X_0Y_0Z_0$ relative to the inertial frame $\xi^*\eta^*\zeta^*$ is known, and the projections $\dot{\Phi}_x$, $\dot{\Phi}_y$, $\dot{\Phi}_z$ of the base absolute angular velocity vector on the system axis $\xi^*\eta^*\zeta^*$ are given functions of time. In addition to the above systems, four more systems of axes $X_BY_BZ_B$, $X_pY_pZ_p$, $X_1Y_1Z_1$, $X_2Y_2Z_2$ are required,

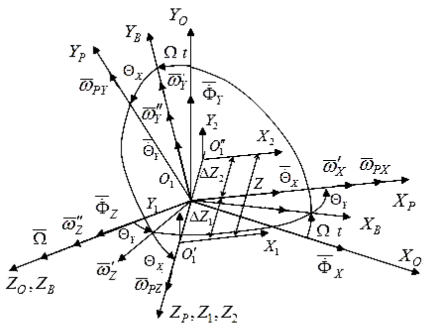


Figure 4. The position of the sensitive element of MMG-N relative to the moving object

connected, respectively, with the drive shaft, with the axes of the main moments of inertia of the rotor and with the corresponding axes of elastic supports (torsion bars), as shown in the figure 4.

The origin of the axes of the $X_B Y_B Z_B$ and $X_p Y_p Z_p$ systems lies at the MMG-N center of mass, and their position relative to the $X_0 Y_0 Z_0$ system is given by successive rotations in the positive direction by an angle Ωt relative to $X_B Y_B Z_B$ and angles Θ_x, Θ_y relative to $X_p Y_p Z_p$ (Ω is angular velocity of the rotor).

The presence of non-crossing of the torsion axes, due to the value of Z, as well as the displacement of the center of mass of the rotor relative to the corresponding axes of elastic supports by ΔZ_1 and ΔZ_2 (figure 4), creates a static imbalance (pendulosity) of the rotor, which allows attributing such a device to the class of "heavy" gyroscopes.

The projections of the angular velocity of the base movement on the system axis $X_B Y_B Z_B$ after turning through the angle Ωt have the form

$$\omega''_x = \dot{\Phi}_x \cos \Omega t + \dot{\Phi}_y \sin \Omega t,$$

$$\omega''_y = \dot{\Phi}_y \cos \Omega t - \dot{\Phi}_x \sin \Omega t,$$

$$\omega''_z = \dot{\Phi}_z + \Omega.$$

The projections of angular velocities on the axis of the system $X_p Y_p Z_p$ after rotation by the angle Θ_y can be written as

$$\omega'_x = \omega''_x \cos \Theta_y - \omega''_z \sin \Theta_y,$$

$$\omega'_y = \omega''_y + \dot{\Theta}_y,$$

$$\omega'_z = \omega''_z \cos\Theta_y + \omega''_x \sin\Theta_y,$$

and after rotation by the angle Θ_x, respectively

$$\omega_{px} = \omega'_x + \dot{\Theta}_x,$$

$$\omega_{py} = \omega'_y \cos\Theta_x + \omega'_z \sin\Theta_x,$$

$$\omega_{pz} = \omega'_z \cos\Theta_x - \omega'_y \sin\Theta_x.$$

Since the MMG-N rotor has a "pendulosity", as a result of its movement with angular velocities ω_{px}, ω_{py} the center of mass moves translationally with a linear velocity (figure 4), projections of which on the system axis $X_pY_pZ_p$ have the form

$$V_{px} = -\omega_{py}\Delta Z_1, \quad V_{py} = -\omega_{px}\Delta Z_2, \quad V_{pz} = 0.$$

To obtain a mathematical model of the gyroscopic system under consideration, we use the Ostrogradsky-Liouville variational principle [12], choosing as generalized coordinates the rotor rotation angles Θ_x and Θ_y, which uniquely determine its position, and as generalized forces: damping moments, elastic moments of torsion bars and moments caused by inertia forces. The latter, under the condition that the angles Θ_x and Θ_y are small, are defined by the following expressions:

$$M_{ix} = m\Delta Z_2(\dot{V}_y \cos\Omega t - \dot{V}_x \sin\Omega t),$$

$$M_{iy} = m\Delta Z_1(\dot{V}_x \cos\Omega t + \dot{V}_y \sin\Omega t).$$

The linearized mathematical model of MMG-N after factorization and taking into account generalized forces up to the first order of smallness has the form

$$(J+\Delta J)\ddot{\Theta}_x + \mu\dot{\Theta}_x + (k_x + (C - J + \Delta J)\Omega^2)\Theta_x - (2J - C)\dot{\Theta}_y =$$

$$-(J+\Delta J)(\ddot{\Phi}_x \cos\Omega t + \ddot{\Phi}_y \sin\Omega t) - \Omega(C + 2\Delta J)(\dot{\Phi}_y \cos\Omega t - \dot{\Phi}_x \sin\Omega t)$$

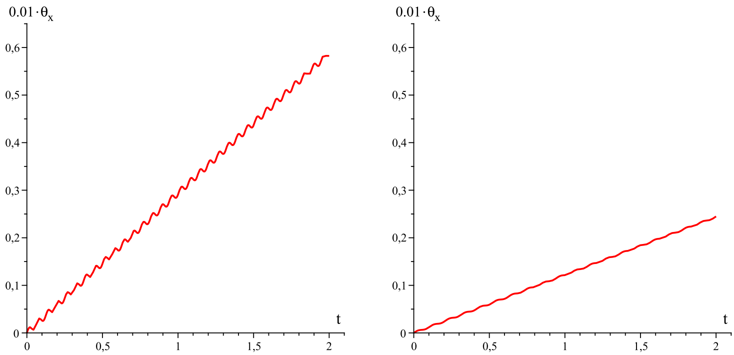
$$+(M+\Delta M)(\dot{V}_y \cos\Omega t - \dot{V}_x \sin\Omega t) + M_x \cos\Omega t + M_y \sin\Omega t, \tag{1}$$

$$(J-\Delta J)\ddot{\Theta}_y + \mu\dot{\Theta}_y + (k_y + (C - J - \Delta J)\Omega^2)\Theta_y + (2J - C)\Omega\dot{\Theta}_x =$$

$$(J-\Delta J)(-\ddot{\Phi}_y \cos\Omega t + \ddot{\Phi}_x \sin\Omega t) + \Omega(C - 2\Delta J)(\dot{\Phi}_x \cos\Omega t + \dot{\Phi}_y \sin\Omega t)$$

$$-(M-\Delta M)(\dot{V}_x \cos\Omega t + \dot{V}_y \sin\Omega t) + M_y \cos\Omega t - M_x \sin\Omega t.$$

The equations (1) are obtained for the case when the MMG-N drive shaft rotates with a constant angular velocity Ω, the stiffnesses k_x, k_y of the torsion bars are small compared to their bending stiffness, and the base acts as an angular movement with speed $\dot{\Phi}_x$, $\dot{\Phi}_y$, $\dot{\Phi}_z$, and translational



(a) angular velocity (b) linear accelerations

FIGURE 5. Solution of the problem in the case of an asymmetric rotor

movement with acceleration $\dot{V}_x$, $\dot{V}_y$, $\dot{V}_z$. The following designations are introduced here:

$$J = \frac{A + B + m(\Delta Z_1^2 + \Delta Z_2^2)}{2}, \ \Delta J = \frac{A - B + m(\Delta Z_1^2 - \Delta Z_2^2)}{2},$$

$$M = \frac{m(\Delta Z_1 + \Delta Z_2)}{2}, \ \Delta M = \frac{m(\Delta Z_1 - \Delta Z_2)}{2},$$

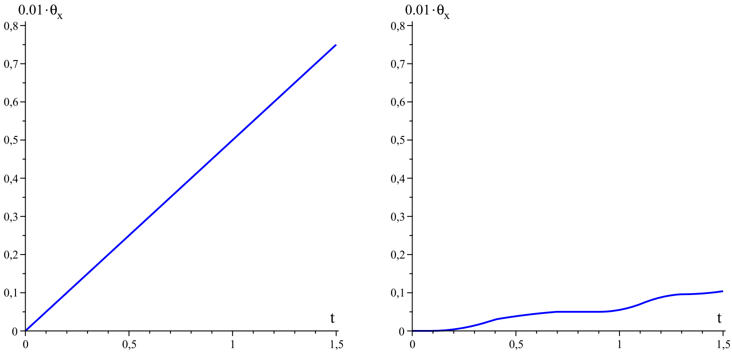
$\mu_x = \mu_y = \mu$ is coefficient of viscous friction, $k_x = k_y = k$ is torsional rigidity of torsion bars, A, B, C are equatorial and polar moments of inertia of the rotor, m is mass of the rotor, M_x, M_y are external moments acting along the corresponding axes of gyroscope sensitivity.

3. Numerical Modelling

The system of equations (1) was solved numerically in the Maple environment under zero initial conditions

$$\Theta_x = \dot{\Theta}_x = 0, \quad \Theta_y = \dot{\Theta}_y = 0. \tag{2}$$

Figure 5 shows the graphs for solving the problem (1)–(2) in the case of an asymmetric rotor ($A \neq B$). Figure 5a corresponds to the channel for measuring the angular velocity then $\dot{\Phi}_y = 0.1$, $\dot{\Phi}_x = 0$ (linear accelerations $\dot{V}_x$ and $\dot{V}_y$ in equations (1) are equal to zero), and Figure 5b corresponds to channel for measuring linear accelerations then $\dot{V}_x = 0.01$, $\dot{V}_y = 0$ (the angular velocity components $\dot{\Phi}_x$ and $\dot{\Phi}_y$ in equations (1) are equal to zero).



(a) angular velocity (b) linear accelerations

Figure 6. Solution of the problem in the case of a symmetric rotor

It can be seen from the graphs in Figure 5 that the use of an asymmetric rotor leads to its precession, accompanied by nutation both in the channel for measuring the angular velocity and in the channel for measuring linear accelerations. The amplitude of nutation oscillations in the case of a high-quality sensitive element is quite large.

Figure 6 shows similar solutions to the problem (1)–(2) in the case of a symmetric rotor with the same equatorial moments of inertia ($A = B$).

A distinctive feature of the graphs in Figure 6 is the absence of nutation in the channel for measuring the angular velocity (Figure 6a, which is associated with the dynamics of the input information of the gyroscope, which responds not only to the angular velocity, but also to the angular acceleration, and if the equatorial moments of inertia of a symmetrical rotor are equal, it is possible to actually suppress its nutation oscillations through the channel for measuring the angular velocity. Along with precession, nutation is observed in the channel for measuring linear accelerations (Figure 6b).

Thus, the precessional motion reflects the response of the gyroscope's sensitive element to the angular and translational motion of the base, while nutational oscillations occur only during its translational motion. Nutation oscillations become informational, since their amplitude and phase contain information about the magnitude and direction of the measured linear accelerations of a moving object.

Note that information on the linear acceleration measurement channel can be obtained only if the factors $(M + \Delta M)$ and $(M - \Delta M)$ in the equations (1) are not equal to zero, i.e. when $\Delta Z_1 \neq 0$ and $\Delta Z_2 \neq 0$ (presence of non-crossing of the torsion axes of the elastic suspension).

4. Conclusion

The authors have constructed a mathematical model of the movement of a modulating micromechanical gyroscope of a hybrid type. Based on the numerical analysis of the simulation results, it is shown that, under certain conditions, the response of the system to the movement of an object has, along with precession, the observed nutation, the parameters of which contain information about the magnitude and direction of the linear accelerations of the moving object. It is noted that the measurement of linear acceleration can be implemented due to a small symmetrical separation of the axes of the elastic suspension relative to the center of mass of the sensitive element.

The results obtained make it possible to solve the important problem of inertial navigation associated with the creation of a multifunctional device that performs the functions of measuring both angular velocities and linear accelerations.

References

[1] L. U. Bakhtieva, V. M. Bogolyubov. "Parametrically excited microelectromechanical system in the problems of orientation of moving objects", *Journal of Physics: Conference Series*, **1158**:2 (2019), 022018, 7 pp. doi ↑18

[2] L. U. Bakhtieva, V. M. Bogolyubov. "Modulation of damping in the rotor vibratory gyroscopes", *Russian Aeronautics (Iz. VUZ)*, **61**:4 (2018), pp. 599–604. doi ↑18

[3] L. U. Bakhtieva, V. M. Bogolyubov. "Astatic gyrocompass based on a hybrid micromechanical gyroscope", 2021 IEEE East-West Design & Test Symposium (EWDTS) (10–13 September 2021, Batumi, Georgia), 2021, pp. 1–5. doi ↑18

[4] L. U. Bakhtieva, V. M. Bogolyubov. "Study on parametrically excited microelectromechanical system in the problems of orientation of moving objects", Research Trends and Challenges in Physical Science, vol. **5**, B P International, 2021, ISBN 978-93-5547-183-3, pp. 150–157. doi ↑18

[5] V. V. Matveyev, V. Ya. Raspopov. "Devices and systems of orientation, navigation and navigation on micromechanical sensors", *KhXIV Sankt-Peterburgskaya mezhdunarodnaya konferentsiya po integrirovannym navigatsionnym sistemam* (Sankt-Peterburg, 29–31 maya 2017 goda), ed. V.G. Peshekhonov, "Kontsern "Tsentral'nyy nauchno-issledovatel'skiy

institut "Elektropribor", SPb., 2017, ISBN 978-5-91995-046-2, pp. 264–266 (in Russian).↑18

[6] D. P. Luk'yanov, V. Ya. Raspopov, Yu. V. Filatov. *Applied theory of gyroscopes*, uchebnik dlya vuzov, GNTs RF OAO Kontsern "Elektropribor", SPb., 2015, ISBN 978-5-91995-036-3 (in Russian), 315 pp. URL↑18

[7] S. F. Konovalov, Yu. A. Ponomarev, D. V. Mayorov. "Magnetic compensation of a zero signal in a two-axis hybrid R-R-R-type MEMS gyro", *Vestnik MGTU im. N.E. Baumana. Ser. "Priborostroyeniye"*, 2013, no. 4(93), pp. 122–131 (in Russian). URL↑18

[8] I. E. Lysenko, D. Y. Sevostyanov, N. F. Kidyaev, A. V. Kovalev. "Research of dynamic characteristics of a three-axis micromechanical gyroscope-accelerometer", *Journal of Physics: Conference Series*, **2086** (2021), 012206, 4 pp. doi↑18

[9] S. Li, L. Chen. "Research on the reliability test of a micromechanical gyroscope with full symmetry structure", *Journal of Physics: Conference Series*, **1650** (2020), 022092, 5 pp. doi↑18

[10] I. Ninalalov, I. Merkuryev, O. Kubryak, S. Astakhov. "Methods for improving the accuracy of an autonomous orientation and navigation system based on micromechanical gyroscopes and optoelectronic sensors", 2022 29th Saint Petersburg International Conference on Integrated Navigation Systems (ICINS) (30 May 2022–01 June 2022, Saint Petersburg, Russian Federation), pp. 1–2. doi↑18

[11] Y. Li, Q. Tan, J. Wen, L. Song, F. Yang. "Study of nonlinear vibration of resonant gyroscope based on uncertainty analysis method", *Microelectronic Engineering*, **263** (2022), 111845. doi↑18

[12] L. U. Bakhtiyeva, F. Kh. Tazyukov. "Solving the stability problem for a thin shell under impulsive loading", Uchen. zap. Kazan. un-ta. Ser. Fiz.-matem. nauki, vol. **156**, 1, Izd-vo Kazanskogo un-ta, Kazan', 2014, pp. 5–11 (in Russian). ↑22

Received 16.08.2022;
approved after reviewing 20.09.2022;
accepted for publication 20.09.2022.

Recommended by *prof. A. M. Elizarov*

Information about the authors:

Lyalya Uzbekovna Bakhtieva

Ph.D. (Physics and Mathematics), Associate Professor, Associate Professor of the Department of Applied Mathematics and Artificial Intelligence, Kazan Federal University. Research interests include mathematical modeling of physical processes

0000-0003-0813-9042

e-mail: lbakhtie@yandex.ru

Vladimir Mikhailovich Bogolyubov

D.Sc. (Engineering), Associate Professor, Professor of the Department for Radio-Electronic and Telecommunication Systems of Kazan National Research Technical University named after A.N. Tupolev. Research interests include radio electronics, control systems, inertial navigation

0000-0001-5014-6305

e-mail: bvm200@yandex.ru

Maxim Dmitrievich Tumakov

1st year M.S. student of ICM&IT of KFU. Research interests include image processing, high-performance computing, mathematical modeling

0000-0002-8028-6939

e-mail: madtumakov@stud.kpfu.ru

The authors contributed equally to this article.

The authors declare no conflicts of interests.