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Calculation of interrelated thermal processes in a submersible electric motor, rocks and water-gas-oil flow in a producing well

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Abstract. This paper is devoted to the study of interrelated thermal processes in a submersible electric motor of a pumping unit located in an oil-producing well and flowed around by a water-oil-gas reservoir mixture, taking into account the heat exchange of the flow with the rocks surrounding the well. To describe these processes, mathematical and numerical models are developed. The numerical model and algorithms are implemented in a software that allows to study temperature fields and various thermal effects using computational experiments with simultaneous visualization of the results of computations. It is shown, in particular, that the transient thermal processes in the system "motor — three-phase flow – rocks", when the motor is turned off due to its heating to the maximum permissible temperature depend on the physical and geometrical characteristics of each element of the system and are characterized by a non-trivial temperature profiles in rocks. Calculated estimates of the duration (on the order of tens of minutes) of the cooling stage of the motor after it is turned off and its heating stage when it is turned on again correspond to the real times of these processes in producing oil wells.

Key words and phrases: mathematical modeling, finite-difference method, computer simulation, thermal processes, heat exchange, oil-gas-water mixture, oil-producing well, surrounding rocks, submersible pumping unit, electric motor, computational experiments

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Introduction

Thermal processes in an oil producing well equipped with a submersible electric centrifugal pump are caused by the interaction between the submersible electric motor (SEM) of the pumping unit, water-oil-gas flow in the annular gap between the motor, well casing and surrounding rocks. Each of these elements is characterized by its own structure, geometrical and physical parameters. A specific feature of such processes is the significant heterogeneity of the three-phase mixture that flows around the motor and then enters the inlet of the pump. The physical and chemical properties (density, viscosity, heat capacity, etc.) of oil, water and associated gas moving along the well from the oil reservoir and forming the water-oil-gas mixture have a significant impact on both the thermal regime of SEM and the interaction of the flow with rocks [1–3]. If one chooses the wrong type of pump unit without taking into account the combined influence of all these factors on the motor operation, a situation, when the flow rate of the three-phase mixture becomes insufficient to cool SEM and as a result, the motor is impermissibly overheated, may arise [1, 4–6].

Surface control stations that have a feedforward and a feedback with pumping units allow avoiding such accidents. Their interaction is based on the analysis of incoming telemetry data (temperature, pressure, etc.) and the generation of necessary actions to improve both the steady-state and transient operation regimes of submersible equipment, up to its automatic shutdown in emergency situations, which, of course, leads to the termination of oil production [1].

In particular, the thermal processes are very important during the commissioning of the wells after underground equipment repair or their transition into a new quasi-stationary operating regime [5, 7, 8]. In these cases, the thermo-hydrodynamic processes inside the well and the pumping unit are non-steady, and the motor can operate for a long time under overload or underload conditions with insufficient cooling by the flow of reservoir fluids. At the stage of commissioning of the well, the control station can repeatedly turn on and off the pump unit to cool the motor with the multiphase flow.

Therefore, the calculation of thermal processes and forecasting of the performance of underground equipment in acceptable temperature ranges are necessary not only to improve the reliability of its operation, but also to optimize oil production from wells. The solution to these problems can be effectively found on the basis of mathematical and numerical modeling.

Thus, the present paper is devoted to the study of the unsteady interrelated thermal processes in the system “SEM — three-phase flow – rocks” using computer simulation.

Note also that the results of our research are intended to be used in cyber-physical systems [7] designed for computer simulation of real technological operations performed by specialists in the oil production.

1. Mathematical Model

As a rule, the temperature regime of the electric motor is calculated for steady-state operating conditions, which are most important for the practice of oil production of the submersible installation [1]. In this case, inertial effects can be neglected, and the coefficient of non-stationary heat transfer can be used to calculate the heat exchange of a multiphase borehole flow with rocks [8].

However, in the study of transient processes in the system “SEM — three-phase flow – rocks”, it is necessary to take into account temporary temperature changes in each of its elements. To calculate the interrelated non-stationary processes of thermal conductivity in a submersible electric motor and rocks, as well as convective heat transfer in a water-oil-gas flow moving in an annular gap between the SEM and the inner surface of the well casing, we have developed a mathematical model that includes three blocks of equations. The main equation of the first block is the equation of non-stationary convective heat transfer

$$V_a C_\rho^* \frac{\partial T}{\partial \tau} = -S_c \alpha_c \left(T - \theta_w\right) + S_m \alpha_m \left(T_m - T\right) - 2C_P^* \left(T - T_I\right), \tag{1}$$

$$C_\rho^* = \sum_{i=1}^{3} \rho_i \varphi_i C_{Pi} + \frac{\rho_1 \varphi_1 L}{1 - C_s F} \frac{\partial \left(C_s F\right)}{\partial T},$$

$$C_P^* = \sum_{i=1}^{3} G_i C_{Pi} + \frac{G_1 L}{1 - C_s F} \frac{\partial \left(C_s F\right)}{\partial T}.$$

This equation describes the change in the average temperature $T(t)$ of a three-phase mixture moving at an average speed w to the pump intake along the annular gap with a volume of V_a and an area of S_a between the cylindrical side surfaces of the well casing and the motor with radii R_c, R_m and the areas S_c, S_m, respectively.

Equation (1) is obtained as a result of approximate integration of the one-dimensional energy equation of the water-oil-gas mixture [8–10] along the length of the annular gap. It is assumed in this integration that due to the small length ($\sim$ 10 m) of the annular gap, the mixture moves along the SEM with the same speed of all phases ($w_i = w$, i = 1,2,3) as a quasi-homogeneous medium with effective properties (such as density $\rho = \rho_1\varphi_1 + \rho_2\varphi_2 + \rho_1\varphi_3$, dynamic viscosity μ, etc.) at the given values of mass flow rates of phases $G_i = \rho_i\varphi_i w S_a$, i=1,2,3 of the mixture $G = G_1 + G_2 + G_3$, pressure P_I and temperature T_I at the motor inlet at point $z = z_I$. Here

t denotes time;

$T_m(t)$ *and* $\theta(r,t)$ are the average temperatures of the motor and the rocks;

$\theta_w(t) = \theta(r,t)|_{r=R_c}$ is the temperature of the well casing wall;

α_c *and* α_m are the local heat transfer coefficients for this wall and the side surface of the motor;

subscripts $i = 1, 2, 3$ denote the parameters of the oil, gas and water phases, respectively;

ρ_i, φ_i, α_{Pi}, α_{Ti} *and* C_{pi} are the average density, volumetric concentration, coefficients of thermal expansion, volumetric elasticity and specific isobaric heat capacity of the i-th phase;

L is the latent heat expended on the phase transition of the gas dissolved in oil to the free state when the pressure decreases below the pressure $P_s(T)$ of oil saturation with gas, as well as in the opposite direction at $P > P_s$.

Note that under such assumptions, the change in temperature and phase concentrations in the annular gap area is determined both by the thermal interaction of the mixture with the motor and rocks, and by the speed of its movement, phase transitions and thermal expansion of phases. The pressure $P(z,t)$ is the same in all phases and equal to P_I along the annular gap area.

Equation (1) of a three-phase mixture energy is supplemented with numerous relationships to calculate the most important characteristics of individual phases and the mixture as a whole, the process of oil degassing, as well as the volume concentrations of phases in the flow:

$$\rho_1 = \frac{\rho_{1,R}\rho_{1,\partial}\,(1 - C_s F)\,[1 - \alpha_{P1}\,(T - T_R) + \alpha_{T1}\,(P - P_R)]}{\rho_{1,\partial}\,(1 - F) + \rho_{1,\partial} F\,(1 - C_s)\,[1 - \alpha_{P1}\,(T - T_0) + \alpha_{T1}\,(P_0 - P_R)]},$$

$$\rho_2 = \frac{MP}{848 g Z_\Gamma T}, \qquad \rho_3 = \rho_{3,R}\,[1 - \alpha_{P3}\,(T - T_R) + \alpha_{T3}\,(P - P_R)]\,,$$

$$C_s = \frac{\rho_2^0 V_0(T)}{\rho_{1,R}}, \qquad C = \frac{C_s(1 - F)}{1 - C_s F}, \qquad w = \frac{G}{\rho S_a},$$

$$\varphi_2 = \frac{G_2\rho_1\rho_3}{\rho_3\,[\rho_1 G_2 + \rho_2\,(G - G_2)] + G_3\rho_2\,(\rho_1 - \rho_3)},$$

$$\varphi_3 = \frac{G_3\,(\rho_1\,(1 - \varphi_2) + \rho_2\varphi_2)}{\rho_3\,(G - G_3) + \rho_1 G_3}, \qquad \varphi_1 = 1 - \varphi_2 - \varphi_3.$$

Here

$\rho_{1,\partial}$ *and* ρ_2^0 are the density of degassed oil and gas under normal conditions ($P = P^0 = 0.1013$ MPa, $T = 273$ K);

$\rho_{1,R}$ *and* $\rho_{3,R}$ are density of oil and water at reservoir pressure P_R and temperature T_R;

$F(P,T)$ is the relative gas factor describing the mass transfer between oil and gas phases; the value of F is defined as the ratio of the mass of gas released from the oil phase at certain (P,T)-conditions and the total amount of gas that is initially dissolved in oil phase at reservoir (P_R,T_R)-conditions; $F(P,T) = \Phi(P/P_s) + \delta_F(P/P_s)\,[T - T_R]$, where

- $\Phi\,(P)$ is the function $F(P,T)$ at the characteristic temperature value of $T = T_R$;

- $\Phi(P/P_s)$ *and* $\delta_F\,(P)$ are determined with empirical relationships constructed in [10] on the basis of the processing of experimental data on the degassing of deep oil samples;

C_s is the mass concentration of gas dissolved in oil under reservoir conditions (P_R,T_R) at $P_R > P_s(T_R)$;

C is the mass concentration of gas in the oil phase in the three-phase mixture moving in the well at $P < P_s(T)$;

$V_0(T)$ is the normal volume of gas released from the oil solution at temperature T and pressure $P = 0$;

Z_Γ is the super-compressibility factor of the real gas;

M is the molecular weight of the gas (methane).

A detailed description of all semi-empirical dependencies, approximation formulas and calculation relationships necessary to close equation (1) can be found in [**7**, **8**, **10**]. The second block contains an equation for the average temperature of the electric motor $T_m(t)$, generalizing the calculation scheme [**1**, **2**]. This equation is obtained under the assumption of uniformity of the material and the cylindricity of the SEM as a result of averaging a three-dimensional thermal conductivity equation with a distributed heat source caused by the energy losses during the motor operation taking into account its heat exchange with a flowing three-phase mixture.

$$C_m V_m \frac{\partial T_m}{\partial \tau} = -S_m \alpha_m (T_m - T) + (1 - \eta_m) N_m, \qquad t > 0. \tag{2}$$

Here C_m is the coefficient of the effective volumetric heat capacity of the motor material; N_m and η_m are the power consumption and efficiency of SEM; V_m is the volume of the motor.

The third block of equations describes the process of radial thermal conductivity in homogeneous rocks surrounding the well casing:

$$\frac{\partial \theta}{\partial t} = a \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial \theta}{\partial r} \right), \quad t > 0, \qquad R_c < r < \infty, \tag{3}$$

$$\theta\big|_{r\to\infty} = T_0, \qquad -\lambda \frac{\partial \theta}{\partial r}\bigg|_{r=R_c} = \alpha_c (T - \theta_w), \tag{4}$$

$$\theta(r,0) = T_0, \qquad R_c < r < \infty, \qquad T_m(0) = T(0) = T_0, \tag{5}$$

where a and λ are the coefficients of heat diffusivity and thermal conductivity of rocks; T_0 is a given value of the initial temperature of the system "SEM–well—rocks".

The heat transfer coefficient α_m on the side surface of SEM is defined as a function $\alpha_m = 0.5 Nu \cdot \lambda^* / (R_c - R_m)$ of the Nusselt number $Nu = c\mathrm{Re}^m \,\mathrm{Pr}^n \,(\mathrm{Pr}/\mathrm{Pr}_w)^\kappa$ according to the generally accepted semi-empirical formulas [**11**, **12**] for laminar and turbulent flows of the mixture in the pipe, where $\mathrm{Re} = 2w(R_c - R_m)\rho/\mu$ and $\mathrm{Pr} = \mu C_P^*/\lambda^*$ are the Reynolds and Prandtl numbers, C_P^* and λ^* are the coefficients of the effective heat capacity and thermal conductivity of the mixture, respectively.

The effective viscosity μ of a dispersed three-phase flow is calculated taking into account its dependence on the concentration of dispersed phases in the water-oil-gas flow according to the formulas given in [8, 10].

For laminar flow at Re<2300: c=0.15, m=0.33, n=0.43, κ=0.25; for turbulent flow at Re>10000: c =0.021, m=0.8, n=0.43, κ=0.25. In the transitional flow regime $Nu = K_0\,(\mathrm{Re}) \cdot \mathrm{Pr}^{0,43}\,(\mathrm{Pr}/\mathrm{Pr}_w)^{0,25}$, where $K_0\,(\mathrm{Re}) = 6.1 \cdot 10^{-3}\mathrm{Re} - 9.461$.

The heat transfer coefficient α_c on the side surface of the well casing is defined in the same way. The initial and boundary conditions (4) and (5) close the system of differential equations (1) – (3). It should be noted once again that this paper provides only some basic relationships, which define the operating parameters of the motor and characteristics of multiphase flows in pipes as well as in rocks. A complete set of special constitutive relations is too big and can be found in our previous publications [8, 10].

2. Numerical Model and Algorithm

The system of equations (1)–(5) is nonlinear and solved numerically. Let us introduce mesh points $t_{j+1} = t_j + h_\tau$ of a uniform grid with the step h_τ along the axis t, $j = 0, 1, 2...$ The differential equations (1) and (2) are approximated by an implicit Euler scheme of the first order $O(h_\tau)$:

$$\begin{aligned} T^{j+1} = T^j + \frac{h_\tau}{V_a C_\rho^{*,j+1}} \big[- S_c \alpha_c^{j+1} \left(T^{j+1} - \theta_w^{j+1}\right) \\ + S_m \alpha_m^{j+1} \left(T_m^{j+1} - T^{j+1}\right) - 2C_P^{*,j+1} \left(T^{j+1} - T_I\right) \big], \\ T_m^{j+1} = T_m^j + \frac{h_\tau}{C_m V_m} \left[-S_m \alpha_m^{j+1} \left(T_m^{j+1} - T^{j+1}\right) + (1 - \zeta_m) N_m \right]. \end{aligned}$$

To solve this system of equations at each time interval $[t_j,\, t_{j+1}]$, we use the method of successive approximations:

$$\begin{aligned} T^{s+1} = T^j + \frac{h_\tau}{V_a C_\rho^{*,s}} \big[- S_c \alpha_c^s \left(T^s - \theta_w^s\right) \\ + S_m \alpha_m^s \left(T_m^{s+1} - T^s\right) - 2C_P^{*,s} \left(T^s - T_I\right) \big], \end{aligned} \tag{6}$$

$$T_m^{s+1} = T_m^j + \frac{h_\tau}{C_m V_m} \left[-S_m \alpha_m^s \left(T_m^s - T^s\right) + (1 - \eta_m) N_m \right], \tag{7}$$

where s is iteration number, $s = 0, 1, 2...$ The values of the initial approximations $T^{s=0} = T^j$, $T_m^{s=0} = T_m^j$ and $T_w^{s=0} = T_w^j$ at the iteration $s = 0$ are taken from the previous time point t_j. The criterion of terminating the iterative process at some iteration $s = S$ is the condition $\max\left\{\left|T_m^{S+1} - T_m^S\right|, \left|T^{S+1} - T^S\right|\right\} < \varepsilon$, where ε is a given accuracy. When this condition is satisfied, the values of the unknown functions T^{j+1} and T_m^{j+1} are determined, respectively, by the values T^{S+1} and T_m^{S+1} at the last iterative step. The heat conduction equation (3) is solved by the finite difference method. First, let us transform equations (3) and (4) introducing the logarithmic coordinate system $\zeta = \ln(r/R_c)$, $r = R_c e^{\zeta}$. It is easy to show that these equations can be written as:

$$\frac{\partial \theta}{\partial t} = \frac{a}{R_c^2} \cdot e^{-2\zeta} \frac{\partial^2 \theta}{\partial \zeta^2}, \qquad 0 < \zeta < \infty, \qquad t > 0, \tag{8}$$

$$-\lambda \frac{1}{R_c} \frac{\partial \theta}{\partial \zeta}\bigg|_{\zeta=0} = \alpha_c \left(T - \theta_w\right), \quad \theta|_{\zeta \to \infty} = T_0, \quad \theta(\zeta, t)|_{t=0} = T_0, \tag{9}$$

where $\theta_w(t) \equiv \theta_{\zeta=0}(t)$. An approximate solution to this problem can be found in the finite region $\zeta \in [0, \bar{\zeta}]$, where the value $\bar{\zeta} = \ln(\bar{R}/\ R_c)$ corresponds to a sufficiently large distance $\bar{R}$ from the well casing wall, at which there is practically no influence of thermal disturbances on the initial temperature of the rocks, i.e. $\theta_{\zeta=\bar{\zeta}}(t) \approx T_0$. To approximate equation (8) and the boundary condition (9) of the 2nd kind, we introduce the points $\zeta_i = i \cdot h$ of a uniform spatial grid with a step $h = \bar{\zeta}/N$, where N is a given number of points, $i = \overline{0, N}$, and the same time grid $\{t_j\}$ with a constant step h_τ. The finite-deference scheme that approximates these equations at the grid points $\{t_j, \zeta_i\}$ with the order $O\left(h_\tau + h^2\right)$ is written as following:

$$\theta_{i+1}^{j+1} - (2 + \mathrm{A}_i)\theta_i^{j+1} + \theta_{i-1}^{j+1} = -\mathrm{A}_i \theta_i^j, \quad i = \overline{1, N-1}, \tag{10}$$

$$\begin{gathered} \theta_1^{j+1} = \theta_0^{j+1} - h \frac{R_c \alpha_c^{j+1}}{\lambda} \left(T^{j+1} - \theta_0^{j+1}\right) + \frac{h^2}{2} \frac{R_c^2}{a} \frac{\theta_0^{j+1} - \theta_0^j}{h_\tau}, \\ \theta_N^{j+1} = T_0, \qquad \theta_i^0 = T_0, \qquad i = \overline{0, N}, \end{gathered} \tag{11}$$

where $\mathrm{A}_i = h^2 R_c^2 e^{2\zeta_i}/(h_\tau a)$; $\theta_0^{j+1} = \theta_w(t_{j+1})$. The system of algebraic equations (10)–(11) with a tridiagonal matrix is solved by the sweep method, if the temperature of the mixture T^{j+1} is given. When implementing a general iterative process, this value is determined as T^{S+1}.

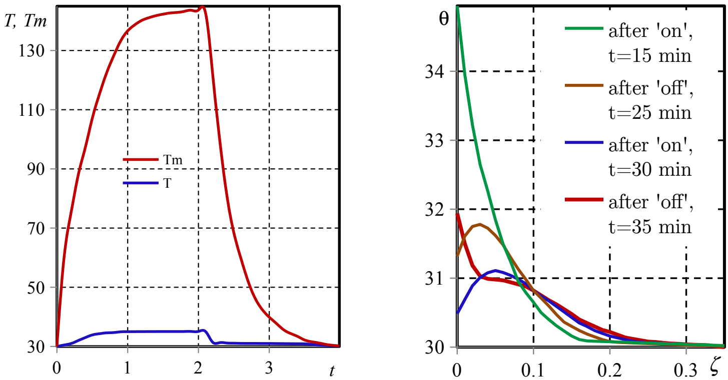
The numerical model and algorithms are implemented in a computer software that allows to carry out the numerical multi-parametric experiments for studying temperature fields and various thermal effects in the producing well, electric motor and rocks, with the simultaneous visualization of the results of computations.

With the use of this software, the stability and convergence of difference schemes were investigated by computational experiment in order to set the optimal grid steps to ensure the necessary accuracy of calculations. A quantitative and qualitative comparison of temperature processes is given for two different approaches to calculating the heat interaction between borehole flow and rocks, the first one of which uses the solution to the problem (3)–(5), while the second one uses the coefficient of unsteady heat transfer obtained on the basis of an asymptotic study of its solution. It was shown that in both cases, the difference between the results of calculations takes place only for small time amounts, whereas for large time amounts, when the transient temperature process tends to the quasi-stationary regime, it practically disappears.

3. Results of Calculations

Let us consider as an example the most interesting from a practical point of view case of thermal processes in the producing oil well that occur when the motor is turned off at the time its temperature T_m as a result of heating reaches the limit value of 130^oC, and when the motor is turned on again after the end of its cooling period, the temperature of the flowing mixture decreases the initial $T_I = 30^oC$. The pressure P_I =5 MPa at the inlet of SEM is lower than the saturation pressure P_s =10 MPa, so that there is free gas in the borehole flow. The volume concentration of water in mixture is 0.5, the total flow rate of the three-phase mixture G=100 m^3/day, the power of the motor, its radius and length are 90 kW, 0.06 m and 12 m, respectively.

The values of numerous parameters of phases and the semi-empirical dependencies of the model (1)–(5) are similar to [**8**, **10**]. The results of calculations for one of the periods of the motor operation and its cooling are shown in Figure 1. As is shown in Figure 1a, starting from the moment t=0, the motor is turned on, its temperature T_m and the temperature T of the water-oil-gas mixture moving in the annular gap



(*a*) $T_m(t)$ and $T(t)$, the time is in hours (*b*) $\theta(\zeta, t)$ after the motor on/off switching

FIGURE 1. The simulation resuts for temperature disturbances into rocks

increase. After about two hours of the motor heating, the temperature T_m reaches the maximum allowable value, and the control station shuts the SEM off. After that, the cooling of motor is beginning with the gas-water-oil mixture. The duration of this stage is about one hour. During this time, gradual equalization of the temperature of SEM to the temperature of the three-phase mixture occurs. After the motor is cooled down, it can resume working again. Figure 1*b* illustrates the changes in temperature profiles and the depth of penetration of temperature disturbances into rocks depending on the time t. From the initial moment $t = 0$ until the motor is turned off, the temperature of rocks and the depth of their heating increase, reaching maximum values by this time. At the cooling stage, there is a gradual decrease in the temperature $\theta(\zeta, t)$, which, due to the inertia of the heat conduction, is significantly delayed in time compared to convective temperature changes in mixture moving rapidly along the annular gap.

Note that the character of the time dependencies of the motor temperature and the duration of the cooling and heating stages of SEM after it is turned off and turned on again (on the order of tens of minutes) agree with the results of field measurements, as well as with similar calculations of the processes in oil wells presented in [**4**, **5**].

4. Conclusions

The mathematical model, algorithms and software for calculating the non-steady thermal processes taking place in the system "submersible electric motor — three-phase flow – rocks" are developed.

On the basis of computational experiments, the convergence and stability of the numerical solution of the problem, as well as the features of non-stationary thermal processes in the submersible motor during the commissioning of an oil well are studied.

High speed of computations with the use of the developed software is shown.

The obtained results are used in a computer cyber-physical system for modeling and forecasting the technological processes of oil production, as well as for training specialists working in the oil industry.

An analysis of the results of calculations under different conditions at the inlet of various types of submersible motors showed that

(1) the increase of the share of water in the three-phase mixture flowing around the motor reduces its temperature, and the appearance of gas in the mixture at a pressure below the saturation pressure, in contrast, leads to an opposite effect — reducing the cooling of SEM by the three-phase flow and raising its temperature;

(2) the penetration depth of temperature disturbances during heating of rocks by three-phase mixture depends on the performance characteristics of the motor, the composition of the mixture and its flow rate. Under constant conditions at the pump unit inlet the heating intensity gradually stabilizes over time, and for a large time amount the heat process reaches a quasi-stationary mode;

(3) the use of a high power motor can lead to its significant heating and if its cooling by the three phase flow is not sufficient, the overheating of SEM and its shutdown by the surface station controller can occur;

(4) the transient thermal processes in the considered interconnected system "SEM–three-phase flow–rocks", when the motor is turned off due to its heating to the maximum permissible temperature, depend on the parameters of each of the elements of this system and are characterized by a non-trivial temperature profiles in rocks. Computed estimates (on the order of tens of minutes) of the duration of the cooling stage of the motor after it is turned off and its heating stage, when it is turned on again, correspond to the real times of these processes in oil wells.

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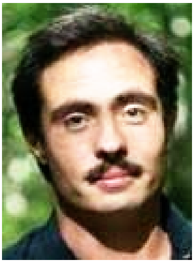

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